Code No. E-10417

FACULTY OF SCIENCE

B.A. / B.Sc. (CBCS) V – Semester Examination, December 2022 / January 2023

Subject: Mathematics

Paper – V : Linear Algebra

Time: 3 Hours **Max. Marks: 80**

PART – A

Note: Answer any eight questions. **(8 x 4 = 32 Marks)**

1. Define a vector space and given an example of vector space.
2. Prove that the intersection of two sub spaces is again a subspace.
3. If $A = \begin{bmatrix} 6 & -4 \\ -3 & 2 \\ -9 & 6 \end{bmatrix}$ then find Null space of A.
4. Find the eigen values of $A = \begin{bmatrix} 6 & 8 \\ 8 & -6 \end{bmatrix}$.
5. Find rank of a matrix having order 4×7 with 4 –dimesnional null space.
6. If λ is an eigen value of an invertible matrix A, then porve that $\frac{1}{\lambda}$ is an eigen value of the matrix A^{-1}.
7. Is every matrix diagonalizable? Mention the condition for the given matrix to be diagonalizable.
8. Find the eigen values and a basis for each eigen space in c^2 for $A = \begin{bmatrix} 1 & -2 \\ 1 & 3 \end{bmatrix}$.
9. Prove than an $n \times n$ matrix with n distinct eigen values is diagonalizable.
10. If $u = [2, -5, -1]^T$ and $v = [3, 2, -3]^T$ then find the inner product of u and v.
11. If u, v are two vectors. Then prove that two vectors, u, v are orthogonal if and only if $\|u - v\|^2 = \|u\|^2 + \|v\|^2$.
12. Prove that, in an inner product space, any orthogonal set of non-zero vectors is linearly independent.

PART – B

Note: Answer all the questions. **(4 x 12 = 48 Marks)**

13. (a)(i) Given V_1 and V_2 in a vector space V. Let $H = span\{V_1, V_2\}$ then show that H is a subspace of V.

(ii) Prove that the null space of an $m \times n$ matrix A is a subspace of R^n.

(OR)

(b)(i) Find a spanning set for the null space of the matrix

$$A = \begin{bmatrix} -3 & 6 & -1 & 1 & -7 \\ 1 & -2 & 2 & 3 & -1 \\ 2 & -4 & 5 & 8 & -4 \end{bmatrix}$$

(ii) Let $\beta = \{b_1, b_2 \dots, b_n\}$ be a basis for a vector space V. Then prove that for each $x \in V$ there exists a unique set of scalars $c_1, c_2 \dots, c_n$ such that $x = c_1b_1 + c_2b_2 + \dots + c_nb_n$.

PART – B

Note: Answer any four questions. **(4 x 12 = 48 Marks)**

13. State and prove spanning set theorem.

14. Define nul space and find basis for the nul space of matrix

$$A=\begin{bmatrix}1 & 1 & -2 & 1 & 5\\ 0 & 1 & 8 & -1 & -2\\ 0 & 1 & 0 & -1 & 14\end{bmatrix}$$

15. State and prove rank theorem.

16. Find eigen values and eigen vectors of $A=\begin{bmatrix}4 & -1 & 6\\ 2 & 1 & 6\\ 2 & -1 & 8\end{bmatrix}$.

17. Compute A^6, where $=\begin{bmatrix}4 & -3\\ 2 & -1\end{bmatrix}$ using $A = PDP^{-1}$

18. Construct general solution of $x' = Ax$ where $A = \begin{bmatrix}-3 & 2\\ -1 & -1\end{bmatrix}$.

19. If $S = \{u_1, u_2 \dots\dots u_P\}$ is orthogonal set of non zero vectors in R^n, then prove that S is linearly independent and hence is a basis for subspace spanned by S.

20. Let W be the subspace spanned by the set $S = \{x_1, x_2, x_3\}$ where

$x_1 = \begin{bmatrix}3\\ 1\\ -1\\ 3\end{bmatrix}$ $x_2 = \begin{bmatrix}-5\\ 1\\ 5\\ -7\end{bmatrix}$ $x_3 = \begin{bmatrix}1\\ 1\\ -2\\ 8\end{bmatrix}$. Now Construct an orthogonal basis for W.

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No. 6516

FACULTY OF SCIENCE

B.Sc./ BA V Semester (CBCS) Examination, March 2022

Subject: Mathematics

Paper – V : Linear Algebra

Time: 3 Hours **Max. Marks: 80**

PART – A

Note: Answer any eight questions. **(8 x 4 = 32 Marks)**

1. Determine whether the Set S = $\{v_1, v_2, v_3\}$ is a basis of $\mathbf{R}^3$, where

$$v_1 = \begin{bmatrix} 3 \\ 0 \\ -6 \end{bmatrix} \quad v_2 = \begin{bmatrix} -4 \\ 1 \\ 7 \end{bmatrix} \quad v_3 = \begin{bmatrix} -2 \\ 1 \\ 5 \end{bmatrix}.$$

2. Prove that intersection of two subspaces is again a subspace.
3. Find the dimension of the subspace H spanned by $\begin{bmatrix} 1 \\ -5 \end{bmatrix}, \begin{bmatrix} -2 \\ 10 \end{bmatrix}, \begin{bmatrix} -3 \\ 15 \end{bmatrix}$.
4. If a 7x5 matrix A has rank 2, Find dim Nul A,
5. If $\begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$ an eigen vector of $\begin{bmatrix} -4 & 3 & 3 \\ 2 & -3 & -2 \\ 1 & 0 & -2 \end{bmatrix}$ then, find eigen value.
6. Find the characteristic polynomial of A = $\begin{bmatrix} 4 & 0 & -1 \\ 0 & 4 & -1 \\ 1 & 0 & 2 \end{bmatrix}$
7. Show that an $n \times n$ matrix with n distinct eigen values is diagonalizable.
8. Let $T: V \to W$ be a linear transformation with $T(b_1) = 3c_1 - 2c_2 + 5c_3$ and $T(b_2) = 4c_1 + 7c_2 - c_3$. Find the matrix M for T relative to bases $B = \{b_1, b_2\}$ and $c = \{c_1, c_2, c_3\}$ for vector spaces V and W.
9. Find the complex eigen values of A = $\begin{bmatrix} 0 & 5 \\ -2 & 2 \end{bmatrix}$.
10. Find a unit vector in the direction of $(1, -2, 2, 0)$.
11. Determine if $\{u_1, u_2, u_3\}$ is an orthogonal set, where $u_1 = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}, u_2 = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}, u_3 = \begin{bmatrix} -1/2 \\ -2 \\ 7/2 \end{bmatrix}$
12. Let $y = \begin{bmatrix} 7 \\ 6 \end{bmatrix}$ and $u = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$. Find the orthogonal projection of y onto u.